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Piecewise Function: It’s about its Domain and Range

Giang-Nguyen Nguyen

The National Council of Teachers of Mathematics Standards for Algebra recommends that “instructional programs from prekindergarten through grade 12 should enable all students to understand patterns, relations, and functions” (2000, p. 297). The function concept in particular is essential to the understanding and learning of algebra. It is the “single most important concept from kindergarten to graduate school” (Dubinsky & Harel, 1992, p vii.). Students at the college level continue to encounter difficulties with the concept of function (Carlson & Oehrtman, 2005). In a college algebra course, students are introduced to various functions; the piecewise function is one concept with which students struggle. This paper describes an in-depth analysis of students’ work on piecewise functions through a single chosen case, an undergraduate named Wendy. Her work illustrates a common incomplete understanding that was exhibited by other students in this study. Task-based interviews were taken from 26 students enrolled in different sections of the course.

The Case of Wendy: Piecewise Functions Sample Work

At the time the study took place, Wendy was enrolled in college algebra to fulfill her degree plan requirement for elementary education. It was her second attempt in the course and she had not yet mastered the piecewise function concept. The following tasks were developed and administered to help delineate the misconceptions and lack of understanding of the 26 students. In the paragraphs that follow, Wendy’s frequently, incorrect work on these tasks will be discussed.

Discussion of Task 1: Graph \( f(x) \), if possible: 
\[
f(x) = \begin{cases} 
3, & x \geq 5 \\
\frac{1}{2}x + 1, & x < 3 
\end{cases}
\]

Wendy was unable to graph this piecewise function. She did not consider the constraint on the domain of that particular function. Additionally, she did not attempt to draw the “piece” for the domain where \( x \) is greater than or equal to 5. Specifically, she did not consider the domain for each “piece” in the function. In some cases students neglected the restrictions on function’s domain, and they drew the graph of \( f(x) \) as two different functions that crossed each other, as if the function was a system of equations. Figure 1 shows the graph that Wendy incorrectly created for \( y = \frac{1}{2}x + 1 \). Wendy actually graphed \( y = x + 1 \). This particular graph is similar to the graphs that other students provided.
Task #:

Graph $f(x)$, if possible. 

\[
f(x) = \begin{cases} 
3, & \text{for } x \geq 5 \\ 
\frac{1}{2}x + 1, & \text{for } x < 3 \\ 
2x^2 - 5 & \text{for } x < 3
\end{cases}
\]

Figure 1: Graph a piecewise function

Discussion of Task 2:

Task 2: Given 

\[
h(x) = \begin{cases} 
-3x - 12, & \text{for } x < -6 \\ 
6, & \text{for } -4 \leq x < 5 \\ 
-2x + 5, & \text{for } x \geq 6
\end{cases}
\]

(a) Is $h(x)$ a function? Explain why or why not

(b) For what value(s) of $x$ does $h(x) = -20$?

Wendy repeated the same mistake about the function's domain and range. The mistake was documented in the second task where students were asked to explain if $h(x)$ is a function and to find the $x$-value(s) for a given value of the function (see Figure 2).

Wendy stared that $h(x)$ was not a function. In explaining why $h(x)$ is not a function in part(a), Wendy used -6 as an $x$-value to evaluate the first piece: $-3(-6) - 12 = 6$; then she compared 6 to -6, the domain value she used to evaluate the function, and wrote that $h(x)$ was not a function. Wendy did the same substitution process for all three pieces and she concluded that $h(x)$ was not a function. Moreover, Wendy compared the $y$-value to the $x$-value she used for computation and concluded they were not equal. Wendy's work indicated she had an incomplete understanding of the function's domain and range.

In part (b), Wendy evaluated the function at $x = -20$ again confusing domain and range, she obtained the $y$-value of 48. She then crossed it out when she realized 48 is not less than -6. Then Wendy made another error as she compared -20 with the value of the function, 5. Moreover,
Wendy compared -20 to the domain value, -4. Additionally, she proceeded to commit another error by putting -20 into the third function piece and obtained the value of 45. Wendy then compared the value of 45 to the domain in that piece, noting that 45 ≥ 6. In the end, she circled the answer \( x ≥ -6 \). Wendy does not seem to understand that the 20 is the range (y value) for the function.

\[ a) \text{ is } h(x) \text{ a function? Explain why or why not.} \]
\[ \begin{align*}
-3(-4) - 12 &= 0 \quad \text{NO, NOT a function} \\
12 - 12 &= 0 \\
-4 &\leq 4 \\
5 &\leq 4 \Rightarrow 0 \\
5 - 4 &\leq 0 &= 0 \\
\end{align*} \]

\[ b) \text{ For what value(s) of } x, \text{ does } h(x) = -20? \]
\[ \begin{align*}
-3(-20) - 12 &= 48 \\
48 &\leq 0 \\
120 - 12 &= 48 \Rightarrow 0 \\
-20 &\leq 5 \\
-2(-20) + 5 &= 45 + 5 = 50 \\
-20 &\leq 0 \\
\end{align*} \]

\[ x ≥ -6 \]

Figure 2: Wendy’s work on piecewise function

Results and Discussions

Analysis of Wendy’s work shows that she struggled with piecewise function because she had not mastered the concepts related to: (1) identifying domain and range of functions, (2) transferring from one representation to another and (3) identifying functions that satisfied given constraints. However, it is important to note that Wendy was not the only student who has struggled with these concepts. These difficulties were highlighted in earlier research that investigated high school students’ understanding of function concepts (Markovits, Eylon, & Bruckheimer, 1986). Additionally, we found that formal function notation added to students’ difficulties similar to what Vinner (1983) documented. For example, Wendy interpreted the question asking her to find the domain of \( h(x) \) such that \( h(x) = -20 \) as evaluating \( h(-20) \). Moreover, Wendy incorrectly provided her explanation of why \( h(x) \) is not a function by taking the
value of the function, 5, and putting it in the domain for the function piece to compare \(-4 \leq 5 < 5\) [sic]. Wendy indicated that 5 is less than 5.

Wendy’s work illustrates that students have difficulties mastering function sub-concepts of domain and range, function evaluation, and function notation. Therefore, it is necessary to provide extra resources for students to assist them in college algebra and possibly in future courses. This study confirmed that students were not fluent in the use of the graphical representation of piecewise functions, something they should have mastered in high school (Carraher & Schliemann, 2007).

**Lesson Learned from the Case**

Wendy’s work represents typical student responses of those who are having difficulty understanding the concept of function. Overall, students did not understand piecewise functions because they did not understand the sub-concept of the function’s domain and range. Understanding student difficulties in learning piecewise functions would be valuable to teachers when designing tasks related to function concepts. These results will benefit teachers and mathematics educators in the realm of teaching and research that accentuate our knowledge about student learning to improve teaching.

To move forward, teachers can look at this case and learn that the concept of function’s domain and range must be emphasized. Wendy’s work illustrates a real lack of mastery of some essential sub-concepts. Based on our interviews with the group of 26 students, we recommend that educators should focus their attention on providing a deeper understanding of function’s domain and range, as well as function notation, so that students can have a “complete” understanding of function.

Additionally, what we can take from the results of this study is that when we teach piecewise functions it is necessary to emphatically, emphasize to students any domain restrictions, the range, function notation, and graphs. Students must understand the domain and range in order to be able to correctly draw the graph, as is illustrated in Wendy’s work. There was evidence to suggest that the idea of function was not fully explained when first introduced in college algebra and she did not carry it over from her high school work.

**Practical Implications for Teachers**

This study demonstrates several factors that contribute to the complexity of the understanding of the concept of function. Ideas related to the sub-concepts, such as the type of domains and ranges (Dreyfus & Eisenberg, 1982) also need to be fully delineated. Students’ incomplete understanding of the concept of function should also be discussed in mathematics education courses in order to help prepare future teachers to be better prepared to facilitate student learning. The fundamental goal of this paper was to unpack a student’s understanding of piecewise function in order to better understand how to design tasks that could help students successfully move forward.
References


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Giang-Nguyen Nguyen is an Assistant Professor of Mathematics Education in the Department of Teacher Education and Educational Leadership at the University of West Florida. Her research focused on motivation and mathematics teaching and learning. She is interested in mathematical modeling, function concepts, geometric reasoning, and the use of technology in teaching and learning mathematics.