Common Core State Standards for Mathematical Practice

Model with Mathematics

A scholarly journal for mathematics educators in Michigan
Mathematical Modeling in a Problem-Based Setting

Student: “Why do we have to learn this content?”

Teacher: “It’s very important! It will be on the test.”

The experience of students learning mathematics by memorizing a collection of facts for exams takes away their appreciation for mathematical knowledge and how that knowledge can be applied to the world. The Common Core State Standards for Mathematics (CCSSM) mathematical modeling topic and the practice standard, CCSS.MATH.PRACTICE.MP4: Model with mathematics (CCSSI, 2010) provide guidance for making a connection between two seamlessly separate environments: mathematics and real-life applications. In this paper, we reflect on our experiences implementing modeling activities in an outreach program called Mathways to Success.

BACKGROUND AND CLASSROOM SETTING

The program Mathways to Success: Increasing STEM Career Awareness through Mathematics Inquiry provides K-12 students with opportunities to develop their dispositions toward mathematics through hands-on activities. Mathways to Success is a grant-funded project of which the first author served as the principle investigator. This program engaged students in hands-on activities and afforded pre-service teachers to work under supervision of in-service teachers and faculty mentors. Weekly session’s goals were determined and shared among project staff. In-service teachers and pre-service teachers planned activities for the week and presented these ideas to two faculty members in charge of final approval and feedback before implementing them in the classroom. In the first semester piloting the program, there were four pre-service teachers (1 elementary, 2 middle, and 1 high school), three in-service teachers (1 elementary, 1 middle school, and 1 high school) and four university faculty members working with 30 students. Researchers are two mathematics educators with the help from one student assistant.

Each of the elementary, middle, and high school groups worked with one or two pre-services and one in-service teacher in the classroom. The faculty mentor circulated in different classrooms to provide assistance as needed. We used three classroom settings for three grade bands. Students spent approximately eight weeks on their project (formulating mathematical problem from real-life aspects and building models). The first week of the program was an introduction and the last week of the program was for students to present their models.

Student career interests and aptitudes were assessed to provide teachers with data to suggest including topics that related to students’ interests and needs. The students spent approximately two hours each week for 10 weeks in the program. They worked individually and collaboratively on their own projects with facilitators in the classrooms. These classrooms were equipped with instructional resources including manipulatives and technological tools (e.g., SMART Board, iPads, laptops, graphing calculators, and mathematical software such as Geometer’s Sketchpad® and GeoGebra).

MATHEMATICAL MODELING PROCESS

Students often solve mathematical problems that are already formulated, such as, “Find the volume of a dome with the radius of 30 feet.” This type of problem falls short in context and does not offer a real-world based motivation for solving it; hence, it lacks in authenticity (Palm, 2008; Tran & Dougherty, 2014). In response to this issue, we illustrated the modeling process introduced in the CCSSM with examples. We describe (a) how the real-life problem (real life issues) was formulated into mathematical problems, (b) how the mathematical problems were computed and interpreted, and (c) how the solution is validated either for
reporting or for use in another modeling process (See Figure 1).

1. From Real-Life Problems to Formulate Mathematical Problems

In this stage of the activity, teachers and students negotiated which real-life topics to explore, such as traveling to a foreign country, starting up a company, buying an island, and building a hospital. The classroom chose to focus on buying an island to investigate. Students who shared similar interests formed groups and brainstormed ideas related to the island for further exploration. In the sub-groups, the elementary students focused on building the theme park, the middle and high school students focused on the infrastructures, health clinics, and transportation.

After forming groups, students were working on brainstorming mathematical problems. For example, Kendra, in the medical group, who was interested in studying about herbal medication and snake venom, wanted to develop a model of a research laboratory to conduct experiments. Similarly, Trang, who was interested in studying medicine for curing blindness, wanted to model an eye clinic for the island. The hospital group, Jay, Paul, and Kelly, who wanted to become nurses and physicians focused on building a hospital. Following is an example of the discussion:

Jay: I’m interested in building a hospital.

Paul: What specifically about the hospital you want to focus on?

Jay: Oh, I’d like to build a model of the hospital.

Paul: That’s a good idea. How about we focus on finding the cost? I guess...we need to know how big the hospital is to go further.

Jay: Let’s tackle this problem.

It took quite a lot of time for the students to come up with mathematical problems. Sometimes, the teachers encouraged students to list what they want to focus on and asked them how they would come to solve them. Some mathematical problems generated included:

- How much will it cost to build a hospital?
- How many clinics are needed on the island?
- How much space should be reserved for an airport?
- How many solar cell panels do we need to cover the surface of a hemisphere building?

2. From Formulate to Compute, then Interpret - Solving Mathematical Problems

A full discussion of each activity is out of the scope of this paper, but we will provide a detail of the solution process for the mathematical problem, “How much would it cost to build a hospital?”

In order to calculate the cost to build the hospital (e.g., construction, solar cell panels, room supplies), the group determined that they had to find the surface area, floor area, and hospital volume. To do so, they assumed that the hospital would have the shape of a dome. At the beginning, the group used 100 feet as the radius of the spherical shape for the hospital. They researched and found a web applet for a dome calculator. When using the dome calculator (http://www.desertdomes.com/domecalc.html), they learned that 200 feet was not long enough to build the desired dome
hospital so they adjusted and used 300 feet.

a. First attempt: Assume that all the floors have the same radius

Each student in the group chose a different mathematical concept to explore. Jayden calculated the surface area of the hemisphere to figure out how many solar panels would be needed to cover the outside surface of the dome. Kelly drew floor plans of a nursing station and calculated the scale to model it. Paul thought all floors would have the same area, so he calculated the area of the main floor (with the largest radius) and multiplied it by the number of designed levels (9) to find the total area. At that juncture, one teacher prompted Paul to question whether all floors had the same area, which led to a discussion within the group members. Then, Paul realized he needed to re-calculate the radius for each of the floors.

b. Second attempt: Assume that all the angles from the center are the same

Paul thought that the central angle, ABC (See Figure 2), was

\[ \frac{90}{8} = 11.25 \text{ degrees} \]

because the heights between floors are the same. Hence, the radius of the second floor was

\[ 150 \cos(11.25^\circ) \].

He continued the calculation of diameters of other floors without knowing that those angles were not identical, until the issue was brought up that the diameter gets smaller as they go up. The group again discussed what needed to be done, then they tried the third approach.

c. Third attempt: The correct approach

Figure 3 shows the group’s third approach. Paul knew that the height between two consecutive floors was equal to 17,

\[ r/9 = 150/9 = 17 \]

Using the tangent ratio of the right triangle ABC,

\[ \sin(A) = 17/150, \]

he calculated angle BAC using a graphing calculator, and then used that value to calculate the radius of the second floor, AB, (see Figure 3). During the validation process, students discussed various attempted that they used to solve the problem such as Pythagorean Theorem and Law of Sine. After validating Paul’s calculation, the group used those radii for further calculations.

To calculate the cost of building the multi-story hospital, the group used information from the web (Office Space Guys, 2014) to look up the price per square foot. After students finished their calculations, they built a physical model of the hospital using rolled newspapers and tape (See Figure 4). Building the physical models, took place in the classroom at the last week of the program. However, due to the time constraint, students rolled the papers before class for about 1.5 hours. Teachers also helped with the rolling the papers so that students will have more time “doing mathematics.”

3. From Interpret to Validate - Go Back to the Real-Life Problems or Report

The hospital group came up with the cost to it. However, on-going discussions still occurred in the
One member of the group, Jay, considered what happened if the first two floors of the hospital are square as he normally saw in everyday life. Another student was concerned about the cost of facility. The process of validation helped them think about the realism aspect of their model, so they reformulated the mathematical problems. The dialogue below shows the discussion about the validation process.

Jay: Okay, so you found the surface area of the building, correct?

Paul: Yeah!

Jay: Are we going to have any windows for the hospital?

Paul: Ouch, we need to recalculate the surface area.

The validation process allowed students to delve into other mathematical concepts that they might not have explored. There were several processes of validating and adding in assumptions. At the end of the program, the group of students felt satisfied to report on their results. When we asked students to reflect on their experience, they said that they wanted to explore more as they, “did a lot of research at home before coming here.”

CHALLENGES

1. Support Students in Formulating Mathematical Problems

The process of formulating mathematical problems from students’ initial interests was quite challenging. By knowing their interests through the career surveys, we came to class prepared with types of problems that students might be concerned. In this setting, we did not start with a specific mathematical content area, but generated mathematical problems from students’ interests. Different mathematical concepts were explored based on the students’ level of understanding. During the discussions with the students, we were open to supporting their ideas, and assisted them in formulating their general interests into mathematical problems. We especially directed students toward problems related to the quantities (e.g., cost, areas, volume) based on our best guess of their available mathematical knowledge.

2. Follow Students as They Compute (Solve) Problems and Interpret Results

The mathematical problems were formulated based on students’ interests, and some ideas were new to the teachers, which made it arduous for the teachers to support. As for the teachers’ roles, we came into the classroom with assumptions about students’ level of mathematical knowledge, especially, with the understanding of important mathematics at their grades. The ideas we received from one class session also informed us how to prepare for the following meeting. In particular, we crowd-sourced instructional resources such as graphing calculators and web applets that might be helpful when solving the problems. We tried to monitor student-

Figure 3: Students’ third attempt

Figure 4: Building a model of the dome hospital
reasoning processes and provided students with prompts to assess conceptions or misconceptions.

3. Bring Students Back to Their Real-Life Interests -- Support for the Validation Process

Very often, students feel they are finished when finding a solution to a mathematical problem, and neglect to re-evaluate what they are asked to find. This might be because students often solve mathematical problems in which real-life aspects are not evident for them to consider. Hence, they do not often come back to the real life for re-evaluating their solutions. As we ran this program, we kept reminding students to look back to their original problems of interest for authenticity. We often posed questions that encouraged them to think, such as: (1) Is there anything else we should consider in this situation? (2) Do we need to modify our assumptions to help address the problem of interest? These questions led students to get into the habit of validating the problems they solved before reporting the results. We observed several iterations of the modeling cycles in the program as students were involved in the validation process.

CONCLUSION

The CCSSM emphasized that mathematically proficient students must demonstrate the eight mathematical practices, of which model with mathematics is one. We reflected on our experience of helping students formulate real-life issues to mathematical problems and suggest that:

- A full mathematical modeling classroom is somewhat messy and requires a lot of instructional decision making.

However, it provides students with opportunities to solve problems arising in everyday life or from students’ interests. Students gain experience in the process of making assumptions, formulating ideas, solving problems, and validation as they address the mathematical problems related to their interests. The mathematical problems should be authentic, that is, they should have the potential to happen in real life.

- Building classroom norms where students felt comfortable and were confident to explore their interests, challenge each other, and collaborate to solve problems is vital for the learning. While working on their project, our students negotiated with each other and evaluated others’ work. Teachers’ roles are to foster that environment and allow students the time to explore mathematics.

REFERENCES


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