Beyond Model Formulation: Assessment of Novices Graphing, Interpreting, and Writing About Their Model and Solution

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Operations Research/Management Science educators need to promote learning outcomes related to building, solving, and interpreting a model, and producing associated business communications. In this paper, we present a test instrument that requires an evaluation of a two-decision business scenario in three domains. The first domain is formulating a linear programming model. The second is a graphical solution analysis. The third is describing the problem and interpreting the recommendation in a professional memo. Using this three-domain format in a quiz, we assess the quality of student performance across three domains, i.e., model formulation, graphical analysis, and management interpretation, for multiple problem components. Statistical results of the current study indicate that students struggle more with graphically solving and interpreting the model and its solution than with formulating the model. In the current study, students are least successful in graphing the isoprofit line, graphing the constraints, and describing those constraints in the management interpretation. To our knowledge, these domains have not been previously reported for objective functions or constraints. Our study also shows that many students exhibit deficiencies in the mechanics of writing.

Keywords: assessment; developing analytical skills; developing communication skills; teaching management science; pedagogical research

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1. Introduction

For example, in an undergraduate management science course, students traditionally practice problem solving by learning how to correctly formulate a mathematical model from a word problem and solve it, as illustrated in Figure 1. In a traditional learning activity that requires students to perform in two domains, each student reads a word problem and identifies the required decisions, an objective, and multiple constraints. Students then write a legend for the decisions and formulate an objective function and constraints to develop a mathematical model for the problem. Next, students may graphically solve a two-decision problem to identify the optimal solution. If a problem involves more than two decisions, students use software to solve their mathematical programming formulation and generate output. In this paper, we focus on the introductory two-decision problems in which students can perform graphical solution analysis. As a component of business assignments, authors have proposed adding a proposal (Grinde and Kammermeyer 2003) or adding a memo (Carrithers and Bean 2008, Williams and Reid 2010, Williams et al. 2014). In a management science
course, a three-domain assignment for a two-decision problem may require students not only to formulate a model and graphically solve it but also to write a memo describing the problem and presenting the optimal solution in a business context (Williams and Reid 2010, Williams et al. 2014).

In this article, assessment of student performance in a business scenario is made in three critical domains, i.e., model formulation, graphical solution analysis, and management interpretation in a memo. These three domains may be used in multiple types of assessments, such as homework, quizzes, and tests. Figure 2 illustrates the three-domain format. Initially, a fictitious business scenario is presented in a memo from the boss. In Domain 1, the business scenario is formulated as a two-decision linear program. In Domain 2, the two-decision business model is graphically solved and the optimal solution is identified. In Domain 3, management interpretation consists of a professional business memo describing the two-decision business problem and presenting a recommendation.

Several researchers have studied student performance in the first domain, model formulation. For example, Murphy and Panchanadam (1999) assessed undergraduate student performance in model building using schemas versus not using schemas in an introductory course in production and operations management. They found that students struggled with defining decision variables and developing constraints with or without schemas. Stevens and Palocsay (2004) assessed student modeling performance in undergraduate management science classes using their proposed translation approach that forms intermediate phrases for building constraints versus not using their translation approach. They also found that students struggled with defining decision variables and identifying constraint coefficients. Similarly, for the most commonly missed constraints in their teaching experience, Brown and Dell (2007) present constraint examples that consist of a short verbal description accompanied by the mathematical expression. After providing instruction on common modeling constructs, Dombrovskai and Guzman (2006) assessed the number of iterations that computer science students performed to formulate an executable production model. After Riddle (2010) introduced students to a product mix model with an active learning exercise, she found that many students omitted the non-negativity constraints when they formulated a product mix problem on a subsequent quiz.

Less empirical attention has been directed toward assessing student performance in domains other than mathematical formulation. When Harrod (2009) surveyed students in undergraduate operations management classes, 69% of the students indicated that the graphical solution method was useful for understanding linear programming. Stevens and Palocsay (2004) assessed student performance with a translation approach that required students to first write their model using text phrases that described the measurable quantities and then formulate their model using decision variables. They found that students improved their ability to identify decision variables and constraint coefficients with the translation approach compared to their performance without it. When Williams et al. (2014) assessed student performance on homework assignments that required model formulations and memo writing, they found that the average student scores were uniformly higher in the model formulation domain than the management interpretation domain. However, they did not assess the decision, objective or constraint components for each domain. Riddle and Smith (2008) presented an analytic rubric for formulation, Excel solver analysis, and interpretation of results, but did not provide assessment results from implementation. A gap in the literature is the assessment of student performance across all three domains (as indicated in Figure 2) for each of the components.

To our knowledge, this paper is the first to present an assessment of student performance with a three-domain quiz for the decision, objective, and constraint components in an undergraduate management science course. Next, we describe our research design including demographics and the test instrument. Then we present our assessment of student performance for the various components across three critical domains: model formulation, graphical solution analysis, and management interpretation in a memo. We also present separate results for writing mechanics. We end with the conclusions of the current study, pragmatic implications for OR/MS educators, study limitations, and directions for future research.
Figure 2  The Proposed Three-Domain Format Adds Management Interpretation to the Process Flow for Student Activities That Demonstrate Problem Solving for a Two-Decision Word Problem That Requires a Linear Programming Formulation and Solution Analysis

![Diagram](diagram.png)

2. Research Design

Participants in the study were students enrolled in the Fall 2015 undergraduate management science course taught by the first author at the University of West Florida. Data collection and analysis were approved by the Institutional Review Board. Of the 44 students enrolled in the course, 40 volunteered to participate in the research following invitations distributed with informed consent forms during the first class lecture. Participants were majoring in Management (which included four participants majoring in General Business and Supply Chain Logistics) and Management Information Systems. The analysis is based on a total of 38 participants who completed the test instrument, of which three were juniors (students with 60–89 semester hours) and 35 were seniors (students with 90 or more semester hours, including a minimum of 20 semester hours of course work at the junior/senior level). The student enrollment and number of consenting participants are shown in Table 1 along with demographic data for major, year, and gender.

During the first four weeks of the semester, students were introduced to all three domains illustrated in Figure 2 through lectures, textbook readings (using the textbook offered by Hillier and Hillier 2014), course material examples, and two three-domain homework assignments. An example of a three-domain homework assignment that required students to demonstrate critical thinking in all three of the domains shown in Figure 2 is given in Williams et al. (2014). The students received feedback on their memo writing as well as their models on both of the three-domain homework assignments before the quiz assessment. In the fifth week, students were given a one-hour three-domain quiz.

The three-domain quiz instrument for Fall 2015 is provided in the online appendix (available as supplemental material at http://dx.doi.org/10.1287/ited.2016.0161). On page one of the three-domain quiz, the analytic rubric shows the three assessment domains for the mathematical comprehension as well as an assessment for the writing mechanics. Using the rubric in the appendix, each of three components are defined across the three domains. The “Decisions” are defined as the legend for the model formulation, the axes labels for the graphical analysis, and the brief description of the decisions in the management interpretation. The “Objective” is defined as the objective function for model formulation, the objective function line (i.e., isoprofit line) for graphical analysis, and the objective for management interpretation. The “Constraints” are defined as the constraints for model formulation, the constraints and feasible region for graphical analysis, and the constraints for management interpretation. A copy of the rubric was provided to the students before the quiz. Students were told that partial credit opportunities were possible for the isoprofit line, feasible region, and optimal solution to the graphical analysis and the description of constraints and optimal solution for the management interpretation in the form of half the points available on the rubric per description area.

On page two of the three-domain quiz, each student read a memo and was required to formulate a problem by writing the decision variable definitions, the objective, and the constraints. On page three, the student graphically solved the model that he or she developed on the first page by labeling the decisions on the axes, graphing each constraint, shading the feasible region, graphing at least one isoprofit line, identifying the optimal solution, and calculating the objective function value. On page four, the student...
Figure 3  The Key for the Three-Domain Quiz Shows Three Assessment Domains for the Fall 2015 Undergraduate Management Science Course

Domain 1: Model Formulation

X1 = # of medium eco-shovels to produce and sell tomorrow
X2 = # of large eco-shovels to produce and sell tomorrow

MAX 1.2X1 + X2  Maximize net revenue ($)
subject to
(1) 0.5X1 + X2 ≤ 90 Switchgrass-based polymer supply (kg)
(2) 4X1 + 4X2 ≥ 480 Labor contract minimum (min)
(3) X1 − 2.5X2 ≤ 0 Marketing # med ≤ 2.5 # large (eco-shovels)
(4) X1, X2 ≥ 0 Non-negativity

Domain 2: Graphical Analysis

Domain 3: Management Interpretation

MEMO TO: Ely Optimizer, Production manager
FROM: Student’s Name
RE: Tomorrow’s production plan for medium and large eco-shovels

In response to your query to determine the number of medium and large eco-shovels to produce tomorrow, I developed a linear programming model to maximize net revenue. I assumed all eco-shovels produced are sold. I ensured non-negative production quantities. My model used at least 8 hours of labor due to the worker contract and at most 90 kg of switchgrass-based polymer due to the quantity available. Per Marketing’s requirement, my model limits the number of medium eco-shovels to at most 2.5 times the number of large eco-shovels. After solving with graphical analysis, I recommend production of 100 medium eco-shovels and 40 large eco-shovels to generate $160 in net revenue.

wrote a memo describing the decisions, stating the objective, and describing the constraints as well as interpreting the optimal solution with a recommendation and indicating its impact on the objective. On page five, an optional rough draft memo space was provided for students.

Figure 3 illustrates the assessment domains with the key for the three-domain quiz for the Fall 2015 undergraduate management science course. For Domain 1, the first two lines show the two business decisions to determine the number of medium and large eco-shovels to produce followed by the objective to maximize net revenue. Next, the mathematical constraints include the limited polymer supply, the labor contract minimum, the marketing forecast ratio constraint, and the non-negativity requirements for the decision variables. For Domain 2, the graphical analysis requires that the student label the axes with the decisions, graph each...
of the four constraints and at least one is profit line such as $1.2X1 + 1X2 = 120$, shade the triangular feasible region, identify the optimal solution (100, 40), and calculate the objective function value of $160$ for the optimal solution. For Domain 3, the memo requires that the student write a description of the decisions, objective, and constraints, and interpret the solution for the recommendation to produce 100 medium eco-shovels and 40 large eco-shovels based on their graphical analysis. Next, we discuss the assessment of student performance on the three-domain quiz.

3. Results

Overall average student performance for three-domain quiz 1 is given in the second column of Table 2 across the three domains: model formulation, graphical analysis, and management interpretation. Statistical analyses were performed on the average student scores for each domain of assessment, i.e., model formulation, graphical analysis, and management interpretation, using SPSS (IBM Corp. 2013). To compare the effect of different domains on average student performance, we conducted a repeated measures analysis of variance (ANOVA) using the General Linear Model approach. There was a significant domain effect on average student assessment scores, Wilks’s lambda = 0.274, $F_{2,36} = 47.81$, $p < 0.001$. For the post hoc analysis, we used three paired $t$-tests to compare student performance on the three domains. The first paired samples $t$-test indicates a difference between model formulation and graphical analysis, $t_{37} = 8.233$, $p < 0.001$. The second indicates a difference between model formulation and management interpretation, $t_{37} = 3.521$, $p = 0.001$. The third indicates a difference between graphical analysis and management interpretation, $t_{37} = -4.275$, $p < 0.001$.

Because the overall scores in Table 2 indicate that student performance differs by domain, further evaluation of each of the five components: decisions, objective, constraints, solution, and objective function value is carried out across multiple domains. For each of the three domains, average performances for the decisions, objective, and constraints are analyzed. In addition, average student scores for the optimal solution and the objective function value are analyzed for two domains, i.e., the graphical analysis and management interpretation.

Three repeated measures ANOVAs were conducted on average student scores for each of the components across the domains using the General Linear Model approach. There was no significant domain effect on average student performance scores for the decisions component (Wilks’s lambda = 0.990, $F_{2,36} = 0.179$, $p > 0.05$). By contrast, there was a significant domain effect on average student performance scores for the objective component (Wilks’s lambda = 0.675, $F_{2,36} = 8.67$, $p = 0.001$) and the constraints component (Wilks’s lambda = 0.368, $F_{2,36} = 30.96$, $p < 0.001$).

All post-hoc comparisons were made within a domain. Table 2 shows average student performance results in components using paired samples $t$-tests. Because three tests were conducted in each component, significance levels of 1.67% (one-third of 5%) were used to ensure that the overall chance of making a type I error was less than 5%.

Table 2 shows that students performed best on the domain model formulation. Furthermore, students performed better on management interpretation than graphical analysis.

For the three components in Table 2, there were no differences for decisions across the three domains. However, for objective and constraints, students performed equally well on model formulation and management interpretation, and significantly poorer on graphical analysis.

Thus far, the three components, i.e., decisions, objective, and constraints, have been compared across all three domains. An additional two components, solution and objective function value, were compared across the graphical analysis and management interpretation domains by conducting two paired sample $t$-tests and are presented with the means ($M$) and standard deviations (SD). For the optimal solution of the graphical analysis ($M = 0.5066$, $SD = 0.2756$) and of the management interpretation ($M = 0.7303$, $SD = 0.1873$) domains, there was a significant domain effect on average student performance scores, $t_{37} = -7.22$, $p < 0.001$. For

<table>
<thead>
<tr>
<th>Domain</th>
<th>Overall</th>
<th>Components</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Decisions</td>
<td>Objective</td>
<td>Constraints</td>
<td></td>
</tr>
<tr>
<td>Model formulation</td>
<td>0.9063*</td>
<td>0.9561*</td>
<td>0.9286*</td>
<td>0.8684*</td>
<td></td>
</tr>
<tr>
<td>Graphical analysis</td>
<td>0.6789*</td>
<td>0.9342*</td>
<td>0.6737*</td>
<td>0.6250*</td>
<td></td>
</tr>
<tr>
<td>Management interpretation</td>
<td>0.8379*</td>
<td>0.9605*</td>
<td>0.8947*</td>
<td>0.8586*</td>
<td></td>
</tr>
</tbody>
</table>

Note 1: The tests are independent across columns.
Note 2: Within columns, means not sharing a superscript are significantly different, $p < 0.017$.
Note 3: The standard deviations for the decisions column are 0.1142, 0.2374, and 0.1366; for the objective column are 0.1232, 0.4091, and 0.2536; and for the constraints column are 0.1419, 0.2599, and 0.2159.
the objective function value of the graphical analysis ($M = 0.4868$, $SD = 0.2961$) and of the management interpretation ($M = 0.4605$, $SD = 0.2937$) domains, there was no significant domain effect on average student performance scores, $t_{37} = 0.53$, $p > 0.05$.

We explored additional repeated measures analyses controlling for two of the demographic factors included in this study, i.e., major and gender. The conclusions were unchanged. It appears from these results that the demographic variables, major and gender, have no effect on student performance. The absence of an effect based on student major may be due to the fact that the courses of study, management and management information systems (MIS) have significant program overlap.

Assessment is also summarized for the number of completely correct problems in each domain and its components in Table 3. Over 80% of the students could identify both decisions correctly in each of three domains. For the other components, the results between Tables 2 and 3 differ, given that average student performance allows for partial credit. For example, if a student set up the constraint such that the coefficients, variables, and right-hand side were correct, but the operator was incorrect, they received partial credit. By contrast, Table 3 identifies the number of completely correct components across multiple domains for the three-domain quiz. For the graphical analysis, both of the rubric elements for constraints and feasible region had to be completely correct for the constraints component in Table 3. Fewer than 50% of the students had the constraints component completely correct in any of the three domains. Interestingly, students struggled transitioning from describing to formulating the constraints and graphing them. More than 50% of the students correctly formulated the objective function, correctly graphed an isoprofit line, and correctly described the objective function. However, only 16% of the students correctly identified the optimal solution and correctly calculated the objective function value. While ten students had a completely correct formulation, only one student had a completely correct model in all three domains. Thus, a logit analysis for completely correct problem and its components could not be performed due to the lack of completely correct solutions.

Another domain separately assessed for the remaining 25 points shown on the rubric in the appendix was the mechanics of memo writing. For this assessment, memo writing included grammar, punctuation, and spelling. Each grammar, punctuation or spelling mistake cost the student one point until their 15 points, 5 points, and 5 points, respectively, were depleted. The number of completely correct and the average student performance scores are given in Table 4. Only 5 of the 38 students in the sample had a perfect score for memo writing mechanics.

### 4. Conclusions and Pragmatic Implications

Results of the current study for a two-decision variable problem indicate that students were most successful in identifying decisions over the rest of the modeling components. This result was consistent with the findings of Murphy and Panchanadam (1999) who also assessed a two-decision problem. For this study, 26% correctly formulated the two-decision product mix problem, while for the study by Murphy and Panchanadam (1999), only 11% and 17% correctly formulated the two-decision diet and 15-decision transportation problems, respectively. While it is believed that the students in this study benefited from the management interpretation domain, it is difficult to make a direct comparison with the earlier studies, given that the problems evaluated were different. Unfortunately, reporting for most of the studies identified in the literature did not include a combination of subject demographics and detailed information about the pedagogical assessment process. Our hope is that providing this level of reporting in our study will facilitate comparisons across future assessment studies.

To our knowledge, this article presents the first assessment of student performance across three domains in demonstrating quantitative and management interpretation skills. Statistical results of the current study indicate that students struggle more with solving and

<table>
<thead>
<tr>
<th>Component</th>
<th>Model formulation</th>
<th>Graphical analysis</th>
<th>Management interpretation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Decisions</td>
<td>33 (87%)</td>
<td>35 (92%)</td>
<td>35 (92%)</td>
</tr>
<tr>
<td>Objective</td>
<td>27 (71%)</td>
<td>21 (55%)</td>
<td>31 (82%)</td>
</tr>
<tr>
<td>Constraints</td>
<td>10 (26%)</td>
<td>6 (16%)</td>
<td>17 (45%)</td>
</tr>
<tr>
<td>Solution</td>
<td></td>
<td>6 (16%)</td>
<td>6 (16%)</td>
</tr>
<tr>
<td>Objective function</td>
<td></td>
<td>6 (16%)</td>
<td>5 (13%)</td>
</tr>
<tr>
<td>Entire domain</td>
<td>10 (26%)</td>
<td>3 (8%)</td>
<td>3 (8%)</td>
</tr>
</tbody>
</table>
interpreting the model and its solution than with formulating the model. In the current study, students are least successful in graphing the isoprofit line, graphing the constraints, and describing the constraints in management interpretation; these domains have not been reported for objective functions or constraints in the literature cited earlier. The current study also shows that many students exhibit deficiencies in the mechanics of writing.

The current study identifies a weakness in student memo writing skills. Given that many students struggled to describe their constraints and interpret their solution, they need more practice writing about analytical models and subsequent recommendations. The practical implications of our findings are that OR/MS educators have an opportunity to expand their focus beyond model formulation by integrating solution techniques and professional communication requirements into their quantitative courses so that students can develop their management interpretation and writing skills for analytical work.

5. Limitations and Future Research

Before describing research extensions, we point out the limitations of the current study. The sample size in our study was relatively small, 38 subjects, due to the limited class sizes at the first author’s university. We recognize that the assessment results presented here were affected by a multitude of factors including concepts emphasized, teaching content, course materials, and level of problem complexity.

An interesting avenue for future research is to explore why students struggled most with the graphical analysis domain. Is there some aspect of the current educational system that renders graphical analysis more difficult? Further research may make comparisons across multiple sections and years. Another avenue for future research is to extend the current study to include larger, more complex business scenarios with more than two decisions, which would replace the graphical analysis with computer software for analysis of the solution domain. For example, an extension to this work would be to test students across the three domains with a more complex problem such as a four material blending problem that requires computer software for analysis and management interpretation of multiple binding and nonbinding constraints. Building on the earlier work of Dombrovskaja and Guzman (2006) and on the current study, an interesting question for future research is whether writing the management interpretation of the problem and the model helps students improve their ability to formulate and solve a correct model with fewer iterations.

Supplemental Material

Supplemental material to this paper is available at http://dx.doi.org/10.1287/ited.2016.0161.

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