Single versus hybrid time horizons for open access scheduling

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Abstract

Difficulty in scheduling short-notice appointments due to schedules booked with routine check-ups are prevalent in outpatient clinics, especially in primary care clinics, which lead to more patient no-shows, lower patient satisfaction, and higher healthcare costs. Open access scheduling was introduced to overcome these problems by reserving enough appointment slots for short-notice scheduling. The appointments scheduled in the slots reserved for short-notice are called open appointments. Typically, the current open access scheduling policy has a single time horizon for open appointments. In this paper, we propose a hybrid open access policy adopting two time horizons for open appointments, and we investigate when more than one time horizon for open appointments is justified. Our analytical results show that the optimized hybrid open access policy is never worse than the optimized current single time horizon open access policy in terms of the expectation and the variance of the number of patients consulted. In nearly 75% of the representative scenarios motivated by primary care clinics, the hybrid open access policy slightly improves the performance of open access scheduling. Moreover, for a clinic with strong positive correlation between demands for fixed and open appointments, the proposed hybrid open access policy can considerably reduce the variance of the number of patients consulted.

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1. Introduction

Open access scheduling, introduced in the early 1990s, is a just-in-time concept that seeks to schedule both short-notice and routine appointments. While open access scheduling can be applied to services such as accounting, financial planning, real estate, law, and healthcare, this paper was motivated by primary care clinics seeking to schedule their provider capacity more effectively. Over the past 10 years, open access scheduling has been implemented in various types of healthcare practices, with many reports appearing from primary care practices (Herriott, 1999; Murray & Tantau, 2000; Forjuoh et al., 2001; Kennedy & Hsu, 2003; Meyers, 2003; O’Hare & Corlett, 2004; Solberg, Hrosicikoski, Sper- Hillen, O’Connor, & Crabtree, 2004; Armstrong, Levesque, Perlin, Rick, & Shectman, 2005; Bundy, Randolph, Murray, Anderson, & Margolis, 2005). It is reported that open access scheduling reduces healthcare costs by decreasing patient no-shows, while improving clinic resource utilization and physician productivity (Kodjababian, 2003; Mallard, Leakeas, Duncan, Fleenor, & Sinsky, 2004; O’Hare & Corlett, 2004; Pierdon, Charles, McKinley, & Myers, 2004; Bundy et al., 2005). Meanwhile, it has also been shown to facilitate timely and patient-centered care, and to improve patient satisfaction (Herriott, 1999; Murray & Tantau, 2000; Kennedy & Hsu, 2003; Mallard et al., 2004; O’Hare & Corlett, 2004; Pickin, O’Catthain, Sampson, & Dixon, 2004; Bundy et al., 2005; Parente, Pinto, & Barber, 2002; Murray & Berwick, 2003). Yet, the open access scheduling concept, still under development, is far from mature. Reports of implementation failures demonstrate the challenges in implementing open access scheduling (Murray, Bodenheimer, Rittenhouse, & Grumbach, 2003; Mehrotra, Keehl-Markowitz, & Ayanian, 2008). In an open access clinic, the short-notice appointments are called open appointments. However, some appointments are still scheduled weeks in advance, which are commonly called fixed appointments. The just-in-time principle of open access scheduling is to deliver healthcare to patients on the day they request it (Murray & Tantau, 2000; Murray & Berwick, 2003). An interesting research question is whether to define only one short-notice time frame or two short-notice time frames in an open access scheduling system.

While over 50 years of appointment scheduling research has proposed many quantitative models to optimize the parameters for traditional outpatient appointment scheduling (Kaandorp & Koole, 2007; Muthuraman & Lawley, 2008) (see the review by Cayirli & Veral, 2003) or for surgery scheduling (Fei, Meskens, &
To answer this question, we compare two scheduling policies in open access scheduling: one having a single time horizon for open appointments, and the other adopting two time horizons for open appointments.

This paper is organized as follows. In Section 2, we describe a typical open access scheduling policy reported in the literature, and then propose a hybrid open access policy. In Section 3, we discuss two performance measures to compare open access scheduling policies. In Sections 4 and 5, we present the analytical and numerical results of the comparison between the two policies. Finally, conclusions and insights for clinic administrators are discussed in Section 6.

2. Hybrid open access scheduling policy

In open access clinics, some appointment slots are held for short-notice scheduling. Usually, clinic administrators specify the limit for the number of fixed appointments or the lowest percentage of open appointments in a clinic session (typically 4 h). When the number of fixed appointments scheduled in a session reaches the limit before the time horizon for open appointments in the session, patients who request fixed appointments in the session have to choose fixed appointments in other sessions or call again later for an open appointment. In most open access clinics, the time horizon for open appointments is the same day or two days, which is illustrated in Fig. 1a. Therefore, the open access scheduling policy currently used in outpatient clinics specifies a single time horizon for open appointments and the lowest percentage of appointments held for short-notice scheduling. We call this the current open access policy (current OA policy).

(a) Time horizons and demands for fixed and open appointments in the current OA policy

(b) Time horizons and demands for fixed, days-ahead and same-day appointments in the hybrid OA policy

Fig. 1. Time horizons and demands for different types of appointments in the current and hybrid OA policies.
Since the time horizon for open appointments can be as long as several days or even one week in some clinics, we propose a hybrid open access policy (hybrid OA policy), which adopts two time horizons for open appointments and two lowest percentages of open appointments. One time horizon for open appointments is the same day, which means that some open appointments can only be scheduled on the same day. We call these open appointments same-day appointments in this paper. The other time horizon for open appointments is two days or several days (not exceeding a week). Those open appointments scheduled one day or several days ahead are called days-ahead appointments in this paper. Fig. 1b illustrates the time horizons for fixed, days-ahead and same-day appointments in the hybrid OA policy. The hybrid OA policy needs to specify the lowest percentage of days-ahead appointments and the lowest percentage of same-day appointments.

Recent research suggests that continuity of care with a primary care provider has multiple benefits to patient and healthcare systems (De Maeseneer, De Prins, Gosset, & Heyerick, 2003; Notting, Goodwin, Flocke, Zyzanski, & Stange, 2003; Wilson, Rogers, Chang, & Safran, 2005; Rodriguez, Rogers, Marshall, & Safran, 2007). Some clinics group a few physicians as a provider group in an effort to balance the number of appointments that can be scheduled in a session, the demand distribution of fixed and open appointments. For a given N, E(M^F, nF^C) for all nF^C ∈ \{0, 1, . . . , N\} can be calculated by using the recurrence relation

\[ E(M^F, nF^C) = E(M^F, nF^C - 1) - \gamma_f^C |F^F(nF^C - 1)| + (1 - \gamma_o^C) |P^F(nF^C - 1) - F^C(nF^C - 1) - N - nF^C|, \]

where \( \gamma_f^C \) and \( \gamma_o^C \) denote the no-show rates of fixed and open appointments, respectively, \( F^F(*) \) is the cumulative joint probability distribution function of demands for fixed and open appointments, respectively, denoted by \( D^F \) and \( D^C \), respectively, \( P^F(*) \) and \( P^C(*) \) are the cumulative joint probability distribution functions of demands \( D^F \) and \( D^C \), respectively, and \( P^C(*) \) is the probability mass function of demand \( D^C \).

According to the recurrence relation in Eqs. (1) and (2), we can calculate the expected number of patients consulted by a physician in a session, and then find the maximum expectation of \( M^F \), denoted by \( E_{max}(M^F) \), for the corresponding \( nF^C \), denoted by \( nF^C_{max} \). Thus, for the current OA policy, \( nF^C_{max} \) represents the limit of fixed appointments that can be scheduled to maximize the expected number of patients consulted by a physician in a session, and \( E(M^F, nF^C_{max}) = E_{max}(M^F) \).

For the hybrid OA policy, \( qF^C \) and \( qC^C \) denote the lowest percentages of days-ahead and same-day appointments, respectively, to be scheduled with a physician in a session. Thus, the limit of fixed appointments to be scheduled with a physician in a session, denoted by \( nF^C \), is \( |N - (qF^C + qC^C)N| \), and the limit of fixed and days-ahead appointments to be scheduled with a physician in a session, denoted by \( nF^C_{max} \), is \( |N - qF^C N| \). Since \( |N - (qF^C + qC^C)N| \leq |N - qF^C N| \), we know \( nF^C \leq nF^C_{max} \).

In the appendix, we derive the recurrence relations for calculating the expectation and the variance of the number of patients consulted for the hybrid OA policy (\( M^H \)). According to the results in the appendix, we know that the expectation of \( M^H \), denoted by \( E(M^H, nF^C, nC^C) \), is a function of \( N, nF^C, nC^C \), the no-show rates of fixed, days-ahead and same-day appointments, and the demand distribution of fixed, days-ahead and same-day appointments. According to Eqs. (A.1), (A.3), (A.6), (A.9), (A.13), (A.21), (A.22), (A.23) in the appendix, we know that for a given \( N \) and all pairs of \( (nF^C, nC^C) \) satisfying \( 0 \leq nF^C \leq nF^C_{max} \) and \( nF^C + nC^C \leq N \), \( E(M^H, nF^C, nC^C) \) can be calculated by using the recurrence relations

\[ E(M^H, nF^C, nC^C) = E(M^H, nF^C - 1, nC^C) + (1 - \gamma_f^C) |F^F(nF^C - 1)| - (1 - \gamma_o^C) |P^F(nF^C - 1) - F^C(nF^C - 1) - N - nF^C|, \]

and

\[ E(M^H, 0, nC^C) = E(M^H, 0, nC^C - 1) + (1 - \gamma_f^C) |F^F(nC^C - 1)| - (1 - \gamma_o^C) |P^F(nC^C - 1) - F^C(nC^C - 1) - 1 - N - nC^C|, \]
with
\[
E(M^{\text{th}}, 0, 0) = (1 - \gamma_{\text{bs}}^{\text{th}}) \sum_{k=0}^{N - N_k} (N - k)p_k^{\text{th}}(k),
\]
where \(p_k^{\text{th}}(\bullet)\) is the probability mass function of demand for same-day appointments, denoted by \(D_0^s\), and \(\gamma_{\text{bs}}^{\text{th}}\) and \(\gamma_{\text{bs}}^{\text{th}}\) denote the no-show rates of fixed, days-ahead and same-day appointments, respectively. In Eqs. (3)–(5),
\[
G_{\text{th}}^s(n_h^s - 1, n_h^s - n_t^s) = 1 - F_0^s(n_h^s - 1) - F_0^s(n_h^s - n_t^s)
+ F_k^{\text{th}}(n_h^s - 1, n_h^s - n_t^s),
\]
\[
G_{\text{th}}^t(n_h^t - 1, N - n_t^s) = 1 - F_0^t(n_h^t - 1) - F_0^t(N - n_t^s)
+ F_k^{\text{th}}(n_h^t - 1, N - n_t^s),
\]
and
\[
G^s(n_h^s - 1, n_h^s) = \sum_{i=0}^{n_h^s-1} \sum_{j=0}^{n_h^s-i-1} G_{\text{th}}^s(i, j, k).
\]
Here \(F_k^{\text{th}}(\bullet)\) is the joint cumulative probability distribution function of demands for fixed and days-ahead appointments, denoted by \(D_0^s\) and \(D_0^d\), respectively, \(G_{\text{th}}^s(\bullet, \bullet, \bullet)\) is the joint cumulative probability distribution function of demands \(D_0^s, D_0^d\) and \(D_0^t\), respectively; \(G^s(\bullet, \bullet)\) is the joint probability mass function of demands \(D_0^s, D_0^d\); \(G^{\text{th}}(\bullet, \bullet, \bullet)\) is the joint cumulative probability mass function of demands \(D_0^s, D_0^d, D_0^t\).

According to the recurrence relations in Eqs. (3)–(5), we can calculate \(E(M^s, n_h^s, n_t^s)\) for \(0 \leq n_h^s \leq n_t^s \leq N\), and then find the maximum expectation of \(M^s\), denoted by \(E_{\text{MAX}}(M^s)\), and the corresponding pair \((n_h^s, n_t^s)\), denoted by \((n_h^s, n_t^s)_{\text{MAX}}\). Thus, for the hybrid OA policy, \(n_h^s, n_t^s\) represents the limit of fixed appointments to be scheduled that maximizes the expected number of patients consulted, and \(n_h^s, n_t^s\) represents the limit of fixed and days-ahead appointments to be scheduled that maximizes the expected number of patients consulted. Thus \(E(M^s, n_h^s, n_t^s)_{\text{MAX}} = E_{\text{MAX}}(M^s)\).

3.2. Variance of the number of patients consulted

For the current OA policy, according to the results in (Qu, 2006), we know that the variance of \(M^s\), denoted by \(V(M^s, n_h^s)\), is a function of \(N, n_h^s, \gamma_{\text{bs}}^{\text{th}}, \gamma_{\text{bs}}^{\text{th}}, \gamma_{\text{bs}}^{\text{th}}, \gamma_{\text{bs}}^{\text{th}}\), and the demand distribution of fixed and open appointments. For a given \(N\) and all \(n_h^s \in \{0, 1, \ldots, N\}\), \(V(M^s, n_h^s)\) can be calculated by using the recurrence relation
\[
V(M^s, n_h^s) = V(M^s, n_h^s - 1) + Q_{\text{th}}^{\text{th}}(n_h^s - 1),
\]
with
\[
V(M^s, 0) = (1 - \gamma_{\text{bs}}^{\text{th}}) \sum_{k=0}^{N} (N - k)p_k^{\text{th}}(k) - (Q_{\text{th}}^{\text{th}})^2.
\]
Here,
\[
Q_{\text{th}}^{\text{th}}(n_h^s - 1) = (1 - \gamma_{\text{bs}}^{\text{th}})(1 - \gamma_{\text{bs}}^{\text{th}})(1 - \gamma_{\text{bs}}^{\text{th}})[F_0^s(n_h^s - 1) - F_0^s(n_h^s - n_t^s) + 2A_1(n_h^s - 1)],
\]
\[
Q_{\text{th}}^{\text{th}}(n_h^s - 1, N - n_t^s) = (1 - \gamma_{\text{bs}}^{\text{th}})(1 - \gamma_{\text{bs}}^{\text{th}})G_0(n_h^s - 1, \ldots, n_h^s - 1).
\]

4. Analysis of hybrid open access scheduling policy

In Section 3, we discussed two performance measures of open access scheduling, the expected number of patients consulted and the variance of the number of patients consulted. Next, we
compare the current and hybrid OA policies in terms of the two performance measures. The time horizon for open appointments in the current OA policy could be the same day or several days (not exceeding a week). To compare the hybrid OA policy and the current OA policy, we assume that the time horizon for open appointments in the current OA policy is two days. Under the assumption that the two policies do not change the patient demand of a physician, we know $D_0 = D_0^c = D_0^h$, and we have $n^c = n^c_s = n^c_d + n^c_s$ and $n^h = n^h_s = n^h_d + n^h_s$, where $n^c$ and $n^h$ are the numbers of days-ahead and same-day appointments scheduled, respectively, in the hybrid OA policy, and $E(X_d)$ and $E(X_s)$ are the expectations of $X_d$ and $X_s$.

**Proposition 1.** For the current and hybrid OA policies, when $D_0^c = D_0^h$, $D^c_0 = D^c_0^h + D^c_1^h$, $\gamma^c_s = \gamma^c_d$, and $\gamma^h_s = \gamma^h_d + \gamma^h_s$, then $\mathbb{E}(\max(M^h) - \max(M^c))$.

**Proof.** Assuming that $n_{i,\text{max}}^c$ is the limit of fixed appointments to schedule in the current OA policy that maximizes the expected number of patients consulted, $E(M^c)$, $n_{i,\text{max}}^c = E(M^c)$. Since $D_0^c = D_0^h$, $D^c_0 = D^c_0^h + D^c_1^h$, $\gamma^c_s = \gamma^c_d$, and $\gamma^h_s = \gamma^h_d + \gamma^h_s$, we know $E(M^h) - n_{i,\text{max}}^h = E(M^c)$, $n_{i,\text{max}}^h = E(M^h)$ for $n^h_i = n_{i,\text{max}}^c$ and $n^h_i = N$. Since $E(M^h) \geq E(M^c, n_{i,\text{max}}^h, n^h_i)$ for any pair ($n_{i,\text{max}}^c$, $n^h_i$), then $E(M^h) \geq E(M^c, n_{i,\text{max}}^h, n^h_i)$. □

**Proposition 2.** For the current and hybrid OA policies, when $D_0^c = D_0^h$, $D^c_0 = D^c_0^h + D^c_1^h$, $\gamma^c_s = \gamma^c_d$, and $\gamma^h_s = \gamma^h_d + \gamma^h_s$, then $\mathbb{V}(\max(M^h)) \leq \mathbb{V}(\max(M^c))$.

The proof of Proposition 2 is similar to that of Proposition 1. According to Propositions 1 and 2, the hybrid OA policy is never worse than the current OA policy in terms of the maximum expectation and the minimum variance of the number of patients consulted.

**5. Numerical scenarios**

In Section 4, we analytically compare the current and hybrid OA policies. Since there are no closed-forms for $E(\max(M^h))$, $E(\max(M^c))$, $\mathbb{V}(\max(M^h))$, and $\mathbb{V}(\max(M^c))$, we investigate the performance improvement by using the hybrid OA policy over a population of 288 scenarios representative of the possibilities, which consider different total numbers of appointments available ($N$), different no-show rate combinations ($\gamma^c_d, \gamma^c_s, \gamma^h_d, \gamma^h_s$), and different demand distributions.

**5.1. Characteristics of numerical scenarios**

The clinics visited by the authors usually have 4-h clinic sessions with 15-min appointment slots, i.e., 16 appointment slots that can be booked. Since some appointments are scheduled in two successive appointment slots, the number of appointment slots available may be less than 16. Therefore, in the numerical scenarios at most 12 or 16 appointments can be scheduled. The total number of appointments available is also called physician capacity in this paper.

Since fixed appointments can be scheduled weeks in advance, the associated no-show rates can reach as high as 50–55% (George & Rubin, 2003; Lee et al., 2005). It is also reported that the average no-show rate increases from 15% to over 35% when the length of interval from the date an appointment is scheduled to the appointment date increases from 4 weeks to over 10 weeks (Kodjababian, 2003). Meanwhile, the no-show rate of open appointments is generally much lower than that of fixed appointments as reported in the literature. For example, the no-show rate resulting from an open access pilot project for primary care decreased from 16% to 11% (Bundy et al., 2005). The average no-show rate decreased from 31% to 16% after the implementation of open access scheduling in another primary care clinic (Kodjababian, 2003). The no-show rates of 20% and 10% are used for days-ahead and same-day appointments, respectively, in one combination of no-show rates ($\gamma^h_d, \gamma^h_s, \gamma^c_d, \gamma^c_s$) to examine the influence of the no-show rates $\gamma^h_d$ and $\gamma^h_s$ in an undesirable scenario, while the no-show rates of 5% and 2% are used for days-ahead and same-day appointments in a significantly improved scenario. Therefore, the four combinations of ($\gamma^h_d, \gamma^h_s, \gamma^c_d, \gamma^c_s$) tested in the numerical scenarios are (0.4, 0.2, 0.05, 0.02), (0.4, 0.1, 0.05, 0.02) and (0.25, 0.1, 0.05) and (0.25, 0.05, 0.02).

Since the demand for fixed or open appointments is the number of requests for appointments occurring in a given time period, a Poisson distribution is a routine choice for the demand. Unlike the traditional outpatient scheduling systems which postpone a large portion of today’s work into the future, open access clinics schedule appointments for patients on the day that they request to be seen. As a result, demand in a session is typically independent of demands in previous days’ sessions. However, during a demand surge period such as influenza season or when the clinic reopens following a holiday, demands between the sessions may be highly correlated. Therefore, in the numerical scenarios, independent Poisson distributions and a trivariate Poisson distribution are used to capture the distribution of demands for fixed, days-ahead and same-day appointments in these scenarios, respectively. The trivariate Poisson distribution has positive correlation coefficients of $\rho_{D_0, D_0'} = 0.8$, $\rho_{D_0, D_1} = 0.2$, $\rho_{D_0', D_1'} = 0.2$ and $\rho_{D_0, D_1'} = 0.5$. Meanwhile, eighteen combinations of average demands for fixed, days-ahead and same-day appointments, which match with $N$, are considered in the numerical scenarios. Table 1 summarizes the levels of $N$. ($\gamma^h_d, \gamma^h_s, \gamma^c_d, \gamma^c_s$), and the demand distributions for 288 numerical scenarios.

**5.2. Performance comparison between the current and hybrid OA policies**

According to Propositions 1 and 2, we know that the optimized hybrid OA policy is never worse than the current OA policy in terms of the maximum expectation and the minimum variance of the number of patients consulted. Table 2 summarizes the ranges of the performance change by comparing the hybrid OA policy with the current OA policy for different levels of physician capacity, different demand correlations, and different combinations of the no-show rates ($\gamma^h_d, \gamma^h_s, \gamma^c_d, \gamma^c_s$) for fixed, days-ahead and same-day appointments. It can be noted in Table 2 that by using the hybrid OA policy, the maximum expected number of patients consulted increases by at most 10%, and the minimum variance of the number of patients consulted decreases by at most 21.26% in all 288 numerical scenarios. Table 2 reveals that the performance improvement by using the hybrid OA policy, in terms of the maximum expectation and the minimum variance of the number of patients consulted, increases with the increase in the demand correlation between fixed and open appointments, or the increase in the no-show rates of days-ahead and same-day appointments. This implies that the flexibility of the hybrid OA policy makes it easier to control the variability of the number of patients consulted caused by strong demand correlations and higher patient no-show rates. Fig. 2 illustrates the distributions of the changes in the two performance measures by comparing the hybrid OA policy to the current OA policy. Fig. 2a shows that the increase in the maximum expected number of patients consulted by using the hybrid OA policy ranges from 0 to 1%. While Fig. 2b demonstrates that the decrease in the variance of the number of patients consulted is less than 1% in more than half of the numerical scenarios, it also shows that the hybrid OA policy improves the minimum variance of the
number of patients consulted by more than 1% in 35% of the scenarios. While the hybrid OA policy only slightly improves the performance of open access scheduling in terms of the maximum expectation and the minimum variance of the number of patients consulted in most scenarios, there are scenarios in which the improvement is noticeable.

Nine scenarios demonstrated variance may decrease more than 10%. Table 3 shows that all nine scenarios have correlated demands for fixed, days-ahead and same-day appointments, and the lowest average demand for same-day appointments. The results imply that the minimum variance of the number of patients consulted decreases more by using the hybrid OA policy as the positive correlations between demands for fixed, days-ahead and same-day appointments increase. This conclusion is also supported by the data summarized in Table 2. Therefore, for a clinic with strong positive correlation between demands for fixed and open appointments, the hybrid OA policy can reduce the minimum variance of the number of patients consulted considerably.
tation and the minimum variance of the number of patients consulted. Since there are no closed-forms for the maximum expectation and the minimum variance of the number of patients consulted for the current and hybrid OA policies, we investigate the performance improvement by using the hybrid OA policy through representative numerical scenarios. Our numerical results show that in most situations, the hybrid OA policy only slightly changes the performance of open access scheduling in terms of the two performance measures. Therefore, a single time horizon for open appointments should be adopted in most open access clinics. For clinics that plan to implement open access scheduling, starting with a single time horizon may be advised. However, for clinics with strong positive correlation between demands for fixed and open appointments, the proposed hybrid OA policy may be considered because it could reduce the minimum variance of the number of patients consulted significantly in such clinics.

Our analysis and numerical results provide insights for clinical administrators to determine whether two time horizons for open appointments are needed in their clinics. For open access clinics, administrators could calculate the correlation coefficient of historical requests for fixed and open appointments in each session. If the correlation coefficient is high and positive, the proposed hybrid OA policy could be considered; otherwise, a single time horizon for open appointments is advised.

In conclusion, this paper determines when to use a single time horizon or two time horizons for open appointments. Interesting extensions are to determine the best lengths of time horizons for open appointments.

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Appendix A. Recurrence relations for the expectation and variance of the number of patients consulted by using the hybrid open access policy

In this paper, we propose a hybrid open access policy (hybrid OA policy), which adopts two time horizons for open appointments, a lowest percentage of same-day appointments, and a lowest percentage of days-ahead appointments. To compare the hybrid OA policy, we derive the recurrence relations for calculating the expectation and the variance of the number of patients consulted when adopting the hybrid OA policy.

For the hybrid OA policy, let \( N \) denote the number of appointments that can be scheduled, \( n_h^f \) the limit of fixed appointments to be scheduled, and \( n_d^f \) the limit of fixed and days-ahead appointments to be scheduled, with a physician in a clinic session. Use \( D_h^f \), \( D_h^d \) and \( D_h^a \) to denote the demands for fixed, days-ahead, and same-day appointments, respectively, and \( n_h^f \), \( n_d^f \) and \( n_d^a \) to denote the no-show rates of fixed, days-ahead and same-day appointments, respectively.

Let \( M_f, M_d \) and \( M_a \) denote the numbers of fixed, days-ahead and same-day appointments kept, respectively, and \( X_f, X_d \) and \( X_a \) denote the numbers of fixed, days-ahead and same-day appointments scheduled, respectively. \( X_f = \min(D_h^f, n_h^f) \) because if there are more than \( n_h^f \) requests for fixed appointments, only \( n_h^f \) of them are granted. Similarly, \( X_d = \min(D_h^d, n_d^f) \) and \( X_a = \min(D_h^a, N - X_d) \). Thus, \( X_f \) is a function of \( n_h^f \), while \( X_d \) and \( X_a \) are functions...
of \(n_i^0\) and \(n_i^1\). We can explicitly denote them as \(X_i = X_i(n_i^0)\), \(X_d = X_d(n_i^0, n_i^1)\), and \(X_0 = X_0(n_i^0, n_i^1)\). Since \(M_0\), \(M_d\), and \(M_i\) are functions of \(X_i, X_d\), and \(X_0\), respectively, \(M_i\) is also a function of \(n_i^0\), while \(M_0\) and \(M_d\) are functions of \(n_i^0\) and \(n_i^1\). We can explicitly denote them as \(M_0 = M_0(n_i^0)\), \(M_d = M_d(n_i^0, n_i^1)\), and \(M_i = M_i(n_i^0, n_i^1)\).

Since the number of patients consulted \(M_i^0 = M_i + M_0 + M_d\), it is also a function of \(n_i^0\) and \(n_i^1\). Therefore, we denote its expectation as \(E(M_i^0, n_i^0, n_i^1)\) and its variance as \(V(M_i^0, n_i^0, n_i^1)\). Assuming independent patient no-shows, we have

\[
E[M_i(n_i^0)] = (1 - \gamma_i^0)E[X_i(n_i^0)]
\]

\[
E[M_d(n_i^0, n_i^1)] = (1 - \gamma_i^0)E[X_d(n_i^0, n_i^1)]
\]

\[
E[M_i(n_i^0, n_i^1)] = (1 - \gamma_i^0)E[X_i(n_i^0, n_i^1)]
\]

\[
Var[M_i(n_i^0)] = (1 - \gamma_i^0)^2Var[X_i(n_i^0)] + \gamma_i^0(1 - \gamma_i^0)E[X_i(n_i^0)]
\]

\[
Var[M_d(n_i^0, n_i^1)] = (1 - \gamma_i^0)^2Var[X_d(n_i^0, n_i^1)] + \gamma_i^0(1 - \gamma_i^0)E[X_d(n_i^0, n_i^1)]
\]

\[
Var[M_i(n_i^0, n_i^1)] = (1 - \gamma_i^0)^2Var[X_i(n_i^0, n_i^1)] + \gamma_i^0(1 - \gamma_i^0)E[X_i(n_i^0, n_i^1)]
\]

\[
E[M_i(n_i^0)] = E[M_d(n_i^0, n_i^1)] + E[M_i(n_i^0, n_i^1)] = (1 - \gamma_i^0)(1 - \gamma_i^0)E[X_i(n_i^0, n_i^1)]
\]

\[
E[M_i(n_i^0, n_i^1)] = Var[M_i(n_i^0)] + Var[M_d(n_i^0, n_i^1)] + Var[M_i(n_i^0, n_i^1)] + 2Cov[M_i(n_i^0)], M_d(n_i^0, n_i^1)] + 2Cov[M_i(n_i^0), M_i(n_i^0, n_i^1)] + 2Cov[M_i(n_i^0, n_i^1), M_i(n_i^0, n_i^1)]
\]

\[
= (1 - \gamma_i^0)^2Var[X_i(n_i^0)] + \gamma_i^0(1 - \gamma_i^0)E[X_i(n_i^0)] + (1 - \gamma_i^0)^2Var[X_i(n_i^0)] + \gamma_i^0(1 - \gamma_i^0)E[X_i(n_i^0)]
\]

\[
+ (1 - \gamma_i^0)^2Var[X_i(n_i^0)] + \gamma_i^0(1 - \gamma_i^0)E[X_i(n_i^0)]
\]

\[
+ 2(1 - \gamma_i^0)^2Var[X_i(n_i^0)] + \gamma_i^0(1 - \gamma_i^0)E[X_i(n_i^0)]
\]

\[
+ \gamma_i^0(1 - \gamma_i^0)Var[X_i(n_i^0)] + \gamma_i^0(1 - \gamma_i^0)E[X_i(n_i^0)]
\]

\[
+ \gamma_i^0(1 - \gamma_i^0)Var[X_i(n_i^0)] + \gamma_i^0(1 - \gamma_i^0)E[X_i(n_i^0)]
\]

\[
+ \gamma_i^0(1 - \gamma_i^0)Var[X_i(n_i^0)] + \gamma_i^0(1 - \gamma_i^0)E[X_i(n_i^0)]
\]

If only same-day appointments are allowed to be scheduled, i.e. \(n_i^0 = n_i^1 = 0\), then \(X_0 = X_d = X_i = \min(D_i^0, N)\). As a result, \(E[M_i(n_i^0)] = 0, E[M_d(n_i^0, n_i^1)] = 0, Var[M_i(n_i^0)] = 0\), \(Var[M_d(n_i^0, n_i^1)] = 0, Cov[M_i(n_i^0), M_d(n_i^0, n_i^1)] = 0, Cov[M_i(n_i^0, n_i^1), M_i(n_i^0, n_i^1)] = 0\), and \(Cov[M_i(n_i^0, n_i^1), M_i(n_i^0, n_i^1)] = 0\). So, we obtain

\[
E[M_i^0, 0, 0] = (1 - \gamma_i^0)^2Var[X_i(0, 0)] + \gamma_i^0(1 - \gamma_i^0)E[X_i(0, 0)]
\]

\[
= (1 - \gamma_i^0)^2\left\{\sum_{k=0}^{N} (N-k)^2p^k_i(k) - \frac{N^2}{N} \right\} + \gamma_i^0(1 - \gamma_i^0)N\sum_{k=0}^{N} (N-k)p^k_i(k)
\]

where \(p^k_i(\cdot)\) is the probability mass function of demand \(D_i^0\).

If no fixed appointments are allowed to be scheduled, i.e. \(n_i^0 = 0\), then \(X_0 = X_d = \min(D_i^1, N)\). Thus, \(E[M_i(n_i^0)] = 0, Var[M_i(n_i^0)] = 0, Cov[M_i(n_i^0), M_d(n_i^0, n_i^1)] = 0, Cov[M_d(n_i^0, n_i^1), M_d(n_i^0, n_i^1)] = 0\). Therefore, when the limit of fixed and days-ahead appointments to be scheduled changes by 1, the change in the expected number of patients consulted is

\[
E(M_i^0, 0, n_i^1) = E(M_i^0, 0, n_i^1 - 1)
\]

\[
= (1 - \gamma_i^0)^2\left[Var[X_i(0, n_i^1)] - Var[X_i(0, n_i^1 - 2)]\right] + \gamma_i^0(1 - \gamma_i^0)\left[Var[X_i(0, n_i^1)] - Var[X_i(0, n_i^1 - 2)]\right]
\]

and the change in the variance of the number of patients consulted

\[
V(M_i^0, 0, n_i^1) = V(M_i^0, 0, n_i^1 - 1)
\]

\[
= (1 - \gamma_i^0)^2\left[Var[X_i(0, n_i^1)] - Var[X_i(0, n_i^1 - 2)]\right] + \gamma_i^0(1 - \gamma_i^0)\left[Var[X_i(0, n_i^1)] - Var[X_i(0, n_i^1 - 2)]\right]
\]
and

\[
\text{Cov}[X_d(0, n_d^1), X_s(0, n_s^2)] - \text{Cov}[X_d(0, n_d^1 - 1), X_s(0, n_s^2 - 1)]
\]

\[
= \sum_{j=0}^{n_d^1-1} \sum_{k=0}^{n_s^2-1} (N - j - k)p_d^j(j, k) - \sum_{j=0}^{n_d^1-1} (n_d^1 - 1 - j)p_d^j(j)
\]

\[
+ \sum_{j=0}^{n_s^2-1} (n_s^2 - 1 - j)p_s^j(j) - \sum_{j=0}^{n_s^2-1} (n_s^2 - 1 - j)p_s^j(j)
\]

\[
= F_d^0(n_d^1 - 1)(E[X_d(0, n_d^1 - 1)] - N + n_d^1 - 1) - [1 - F_d^0(n_d^1 - 1)] - F_s^0(n_s^2 - 1) + F_s^0(n_s^2 - 1 - N - n_s^2 - 1) + n_s^2(n_s^2 - j)p_s^j(j). 
\]

(A12)

where \(p_d^j(j)\) is the probability mass function of demand \(D_d^j\), \(p_d^j(j)\) is the joint probability mass function of demands \(D_d^{j-1}\) and \(D_d^j\). \(F_d^j(j)\) and \(F_s^j(j)\) are the cumulative probability distribution functions of demands \(D_d^j\) and \(D_s^j\), respectively, and \(F_d^j(j)\) is the joint cumulative probability distribution function of demands \(D_d^j\) and \(D_s^j\).

For a given \(n_d^1\), when the limit of fixed appointments to be scheduled changes by 1, the change in the expected number of patients consulted is

\[
E(M_d, n_d^1, n_s^2) - E(M_d, n_d^1 - 1, n_s^2)
\]

\[
= (1 - \gamma_d^1)^2 \{E[X_d(n_d^1)] - E[X_d(n_d^1 - 1)]\}
\]

\[
+ \gamma_d^1(1 - \gamma_d^1) \{E[X_d(n_d^1)] - E[X_d(n_d^1 - 1)]\}
\]

\[
+ (1 - \gamma_d^1)^2 \{E[X_d(n_d^1)] - E[X_d(n_d^1 - 1)]\}
\]

\[
+ (1 - \gamma_d^1)^2 \{E[X_d(n_d^1)] - E[X_d(n_d^1 - 1)]\}
\]

\[
= \sum_{i=0}^{n_d^1-1} \sum_{j=0}^{n_s^2-1} p_d^i(i, j).
\]

(A13)

and the change in the variance of the number of patients consulted is

\[
V(M_d, n_d^1, n_s^2) - V(M_d, n_d^1 - 1, n_s^2)
\]

\[
= (1 - \gamma_d^1)^2 \{Var[X_d(n_d^1)] - Var[X_d(n_d^1 - 1)]\}
\]

\[
+ \gamma_d^1(1 - \gamma_d^1) \{Var[X_d(n_d^1)] - Var[X_d(n_d^1 - 1)]\}
\]

\[
+ (1 - \gamma_d^1)^2 \{Var[X_d(n_d^1)] - Var[X_d(n_d^1 - 1)]\}
\]

\[
+ (1 - \gamma_d^1)^2 \{Var[X_d(n_d^1)] - Var[X_d(n_d^1 - 1)]\}
\]

\[
= \sum_{i=0}^{n_d^1-1} \sum_{j=0}^{n_s^2-1} p_d^i(i, j, k).
\]

(A14)

where

\[
Q_d^1(n_d^1 - 1, n_s^2) = (1 - \gamma_d^1)^2 \{Var[X_d(n_d^1)] - Var[X_d(n_d^1 - 1)]\}
\]

\[
+ \gamma_d^1(1 - \gamma_d^1) \{E[X_d(n_d^1)] - E[X_d(n_d^1 - 1)]\},
\]

(A15)

\[
Q_d^1(n_d^1 - 1, n_s^2) = (1 - \gamma_d^1)^2 \{Var[X_d(n_d^1)] - Var[X_d(n_d^1 - 1)]\}
\]

\[
+ \gamma_d^1(1 - \gamma_d^1) \{E[X_d(n_d^1)] - E[X_d(n_d^1 - 1)]\},
\]

(A16)

\[
Q_d^2(n_d^1 - 1, n_s^2) = (1 - \gamma_d^1)^2 \{Var[X_d(n_d^1)] - Var[X_d(n_d^1 - 1)]\}
\]

\[
+ \gamma_d^1(1 - \gamma_d^1) \{Var[X_d(n_d^1)] - Var[X_d(n_d^1 - 1)]\}
\]

\[
+ \gamma_d^1(1 - \gamma_d^1) \{Var[X_d(n_d^1)] - Var[X_d(n_d^1 - 1)]\}
\]

\[
+ \gamma_d^1(1 - \gamma_d^1)\{E[X_d(n_d^1)] - E[X_d(n_d^1 - 1)]\},
\]

(A17)

\[
Q_d^3(n_d^1 - 1, n_s^2) = (1 - \gamma_d^1)^2 \{Var[X_d(n_d^1)] - Var[X_d(n_d^1 - 1)]\}
\]

\[
+ \gamma_d^1(1 - \gamma_d^1) \{Var[X_d(n_d^1)] - Var[X_d(n_d^1 - 1)]\}
\]

\[
+ \gamma_d^1(1 - \gamma_d^1) \{Var[X_d(n_d^1)] - Var[X_d(n_d^1 - 1)]\}
\]

\[
+ \gamma_d^1(1 - \gamma_d^1)\{E[X_d(n_d^1)] - E[X_d(n_d^1 - 1)]\},
\]

(A18)

\[
Q_d^4(n_d^1 - 1, n_s^2) = (1 - \gamma_d^1)^2 \{Var[X_d(n_d^1)] - Var[X_d(n_d^1 - 1)]\}
\]

\[
+ \gamma_d^1(1 - \gamma_d^1) \{Var[X_d(n_d^1)] - Var[X_d(n_d^1 - 1)]\}
\]

\[
+ \gamma_d^1(1 - \gamma_d^1) \{Var[X_d(n_d^1)] - Var[X_d(n_d^1 - 1)]\}
\]

\[
+ \gamma_d^1(1 - \gamma_d^1)\{E[X_d(n_d^1)] - E[X_d(n_d^1 - 1)]\},
\]

(A19)

and

\[
Q_d^5(n_d^1 - 1, n_s^2) = (1 - \gamma_d^1)^2 \{Var[X_d(n_d^1)] - Var[X_d(n_d^1 - 1)]\}
\]

\[
+ \gamma_d^1(1 - \gamma_d^1) \{Var[X_d(n_d^1)] - Var[X_d(n_d^1 - 1)]\}
\]

\[
+ \gamma_d^1(1 - \gamma_d^1) \{Var[X_d(n_d^1)] - Var[X_d(n_d^1 - 1)]\}
\]

\[
+ \gamma_d^1(1 - \gamma_d^1)\{E[X_d(n_d^1)] - E[X_d(n_d^1 - 1)]\},
\]

(A20)

\[
E[X_d(n_d^1)] - E[X_d(n_d^1 - 1)] = 1 - F_d^1(n_d^1 - 1),
\]

(A21)

\[
E[X_d(n_d^1, n_s^2)] - E[X_d(n_d^1 - 1, n_s^2)]
\]

\[
= 1 + F_d^1(n_d^1 - 1) + F_d^2(n_d^1 - n_d^1 - 1) - F_d^1(n_d^1 - 1, n_s^2 - n_d^1 - 1).
\]

(A22)

\[
E[X_d(n_d^1, n_s^2)] - E[X_d(n_d^1, n_s^2 - 1)]
\]

\[
= \sum_{i=0}^{n_d^1-1} \sum_{j=0}^{n_s^2-1} p_d^i(i, j, k).
\]

(A23)

\[
Var[X_d(n_d^1)] - Var[X_d(n_d^1 - 1)]
\]

\[
= [1 - F_d^1(n_d^1 - 1)] + 2 \sum_{i=0}^{n_d^1-1} \sum_{j=0}^{n_s^2-1} p_d^i(i, j, k).
\]

(A24)

\[
Var[X_d(n_d^1, n_s^2)] - Var[X_d(n_d^1 - 1, n_s^2)]
\]

\[
= [1 - F_d^1(n_d^1 - 1) + F_d^2(n_d^1 - n_d^1 - 1) + F_d^3(n_d^1 - 1, n_s^2 - n_d^1 - 1)]
\]

\[
\times E[X_d(n_d^1 - 1, n_s^2)] + E[X_d(n_d^1, n_s^2)] - 2n_d^1 + 2n_s^1 - 1.
\]

(A25)

\[
Var[X_d(n_d^1, n_s^2)] - Var[X_d(n_d^1, n_s^2 - 1)]
\]

\[
= E[X_d(n_d^1, n_s^2)] + E[X_d(n_d^1, n_s^2)]
\]

\[
\times \sum_{i=0}^{n_d^1-1} \sum_{j=0}^{n_s^2-1} \sum_{k=0}^{n_s^2-1} p_d^i(i, j, k).
\]

(A26)

\[
E[X_d(n_d^1, n_s^2 - 1)] - E[X_d(n_d^1, n_s^2 - 1)]
\]

\[
= \sum_{i=0}^{n_d^1-1} \sum_{j=0}^{n_s^2-1} \sum_{k=0}^{n_s^2-1} (2n_s^1 + 2n_s^1 - 2j + 1)p_d^i(i, j, k).
\]

(A27)
Cor[XnH(1), XnH(1), XnH(1), XnH(1)] = Cor[XnH(1), XnH(1) - 1], XnH(1) - 1, XnH(1) - 1, XnH(1) - 1, XnH(1) - 1] = \(N - nH\) \(1 - F_{DH}(nH - 1) - F_{DH}(nH - nH - 1) + F_{DH}(nH - 1, nH - 1) - nH\) \(1 - N - nH\) \(1 - j\)

and

Cor[XnH(1), XnH(1), XnH(1), XnH(1), XnH(1), XnH(1) - 1, XnH(1) - 1, XnH(1) - 1, XnH(1) - 1, XnH(1) - 1] = \(N - nH\) \(1 - F_{DH}(nH - 1) - F_{DH}(nH - nH - 1) + F_{DH}(nH - 1, nH - 1) - nH\) \(1 - N - nH\) \(1 - j\)

where \(p^{DH}(\bullet)\) is the probability mass function of demand \(D_{DH}, p^{DH}(\bullet)\) is the joint probability mass function of demands \(D_{DH}\) and \(D_{DH}\) is the joint probability mass function of demands \(D_{DH}\) and \(D_{DH}\). \(P^{DH}(\bullet)\) is the cumulative probability distribution function of demand \(D_{DH}\) and \(P^{DH}(\bullet)\) is the joint cumulative probability distribution function of demands \(D_{DH}\) and \(D_{DH}\).

References


